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AN ANALYSIS OF THE SETS  
OF RANK DISTRIBUTION  
IN AN HIERARCHICAL ORGANIZATION

by

Robert Leo Armacost



# United States Naval Postgraduate School



## THESIS

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September 1970

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An Analysis of the Sets of Rank Distribution  
In an Hierarchical Organization

by

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MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

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## ABSTRACT

A Markov type model for studying rank distribution in an hierarchical organization is examined. Various sets of rank distributions are defined and their properties discussed. A computer aided test is developed for testing a given rank distribution to determine if it is an element of a particular set. For a distribution not in the Steady State set, a test is developed to determine if the distribution can be returned to in  $m$  transitions or steps.



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## I. INTRODUCTION

Several authors have studied models for analyzing rank distributions in an hierarchical organization. Bartholomew [1967] uses Markov models to a great extent in his analysis of various social systems. Thonstad [1968] developed a Markov type model for use with the Norwegian educational system. More recently, Branchflower [1970] utilized a Markov type model to examine the promotion and appointment policies of a university and the resulting rank distributions of faculty. Branchflower assumed the promotion policies were fixed and used his model to predict the faculty distribution in the next time period using the current faculty distributions and actual appointments to each rank. In his analysis, Branchflower defines three sets of rank distributions: (1) an Attainable set, (2) the Maintainable set, and (3) the Containment set. An Attainable set is a set of rank distributions which can be reached from some previously attainable set. The Maintainable set is better known as the Steady State set of rank distributions. The Containment set is the set of rank distributions which can be returned to infinitely often by means of a sequence of appointment policies.

The analysis in Branchflower [1970] leads us to pose several questions about the rank distributions given some promotion policy:

1. Can the present rank distribution be maintained for all time by means of appointment policies alone?



2. Can another given rank distribution be attained from the present distribution in a finite time period, or can the desired distribution never be reached?

3. What are the rank distributions which can be returned to at some time?

In this paper, our purpose is first, to define explicitly the sets of rank distributions of interest associated with the model, and to demonstrate various properties of these sets. Second, we shall use these definitions and properties to develop test procedures to determine if a given distribution is an element of a particular set. This analysis will provide answers to questions 1 and 2, and will permit us to give crude bounds as an approximation to answer question 3.



## II. THE BASIC MODEL AND NOTATION

Consider a discrete time Markov model with  $n+1$  states where state  $j$  is the rank of a member of an hierarchical organization. The set of states can be partitioned into two subsets: the first  $n$  states are transient states and are called active ranks; the second subset is the  $n+1^{\text{st}}$  state which is an absorbing state. Let  $p_{ij}$  be the fraction in state  $j$  at time  $t+1$  that were in state  $i$  at time  $t$ . Then

$$\sum_{j=1}^{n+1} p_{ij} = 1, \quad i=1,2,\dots,n+1.$$

We represent these fractions in a transition matrix

$$Q = \begin{bmatrix} P & w \\ 0 & 1 \end{bmatrix}$$

where  $P$  is an  $n \times n$  matrix,  $[p_{ij}]$ ,  $i,j=1,2,\dots,n$ , and  $w$  is an  $n$  dimensional column vector called the loss vector with  $w_i = p_{i,n+1}$ .  $P$  is called the Promotion matrix and describes the transition process of the active ranks in the organization.

It has the property that  $\sum_{j=1}^n p_{ij} \leq 1$ ,  $i=1,2,\dots,n$  with strict inequality holding for at least one row.

We assume that the organization is in an equilibrium condition, and thus keep the total number in the organization constant. We normalize by assuming this number is one.

Let  $x(t)$  be an  $n$  dimensional row vector,  $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$  where  $x_i(t)$  is the fraction in the  $i$ th rank



at time  $t$ . Note that  $\sum_{i=1}^n x_i(t) = 1$ ,  $x_i(t) \geq 0$ . Let  $f(t)$  be an  $n$  dimensional row vector,  $f(t) = (f_1(t), \dots, f_n(t))$  where  $f_i(t)$  is the fractional number of new appointments to rank  $i$  at time  $t$ ,  $f_i(t) \geq 0$ .

The rank distribution at time  $t+1$  is dependent on the rank distribution at time  $t$ , the Promotion matrix, and the appointment policy at time  $t+1$ . In vector notation,

$$x(t+1) = x(t) \cdot P + f(t+1). \quad (1)$$

The requirement that the total number in the organization be constant requires that the total fraction appointed to the active ranks be equal to the total fraction lost to the

absorbing state:  $\sum_{i=1}^n f_i(t+1) = x(t) \cdot w. \quad (2)$

We use the following notation throughout the remainder of the paper:  $e$  is an  $n$  dimensional vector with all components equal to 1;  $e_j$  is an  $n$  dimensional unit vector with the  $j^{\text{th}}$  component equal to 1. It is apparent by the context whether they are row or column vectors.



### III. SET DEFINITIONS

The set of all rank distributions,  $F$ , is defined as

$$F \equiv \{x; x \cdot e = 1, x \geq 0\} .$$

Note that the time notation is omitted when it does not affect the definition.

An Attainable set is a set of rank distributions which can be reached in one step from a previous set of rank distributions. Let  $m$  be the number of steps (time periods) from the set of all rank distributions,  $F$ , then

$$A_m \equiv \{x; x \geq z \cdot P, x \in F, z \in A_{m-1}\} , m = 1, 2, \dots \text{ and } A_0 = F.$$

Consider the sequence of Attainable sets,  $\{A_m\}$ . If the sequence converges to a set  $C$  we call the limiting set the Containment set. Mathematically, we have

$$\begin{aligned} C &\equiv \lim_{m \rightarrow \infty} A_m \\ &= \lim_{m \rightarrow \infty} \{x; x \geq z \cdot P, x \in F, z \in A_{m-1}\} \\ &= \{x; x \geq z \cdot P, x \in F, z \in C\}, \text{ if the limit exists.} \end{aligned}$$

This definition is somewhat unusual in that the Containment set is defined in terms of itself.

The set of all distributions which can be maintained in successive time periods is called the Steady State set,  $M$ . Let  $f(t) = f$  for every  $t$ , then since  $x(t+1) = x(t)$ , from



(1) we have  $x(t) = x(t) \cdot P + f$

$$x(t) \cdot (I-P) = f \geq 0$$

so we define

$$M \equiv \{x; x \cdot (I-P) \geq 0, x \in F\} .$$

If a rank distribution can be reached from the Steady State set,  $M$ , then we call a set of such distributions a Reachable set. All distributions which are reachable from  $M$  in  $m$  steps are in the set  $B_m$ , where

$$B_m \equiv \{x; x \geq z \cdot P, x \in F, z \in B_{m-1}\} , m = 1, 2, \dots \text{ and } B_0 = M.$$

Now consider the sequence of Reachable sets,  $\{B_m\}$ . If this sequence converges to a set  $B$ , we call the limiting set the Reached set. Mathematically, we have

$$\begin{aligned} B &\equiv \lim_{m \rightarrow \infty} B_m \\ &= \lim_{m \rightarrow \infty} \{x; x \geq z \cdot P, x \in F, z \in B_{m-1}\} \\ &= \{x; x \geq z \cdot P, x \in F, z \in B\} , \text{ if the limit exists.} \end{aligned}$$

Thus by their definitions, we see that the Containment set and the Reached set must be the same set when they both exist. In section IV we prove they always exist.



#### IV. SET PROPERTIES

In this section, we list several properties of the sets defined in section III without proofs. We then prove several lemmas and two Theorems which establish the existence of the Containment set and Reached set.

PROPERTY 1: The set of all rank distributions,  $F$ , is convex and closed.

PROPERTY 2: Any Attainable set,  $A_m$ , is convex and closed.

PROPERTY 3: The Steady State set,  $M$ , is convex and closed.

PROPERTY 4: Any Reachable set,  $B_m$ , is convex and closed.

LEMMA 1: An Attainable set,  $A_m$ , is a subset of every previously attainable set.

Proof:  $F = A_0 = \{x; x \cdot e = 1, x \geq 0\}$

$$A_1 = \{x; x \geq z \cdot P, x \in F, z \in A_0\}$$

Pick  $x \in A_1 \Rightarrow x \in A_0$  therefore  $A_1 \subseteq A_0 = F$

Assume  $A_{m-1} \subseteq A_{m-2} \subseteq \dots \subseteq A_1 \subseteq A_0$ .

Define  $A_m(z) \equiv \{x; x \geq z \cdot P, x \in F\}$ ,  $z \in A_{m-1}$  so

$A_m = \bigcup_{z \in A_{m-1}} A_m(z)$  . Pick  $z^* \in A_{m-1}$ , then since  $A_{m-1} \subseteq A_{m-2}$ ,

$A_m(z^*) = \{x; x \geq z^* \cdot P, x \in F, z^* \in A_{m-2}\}$ , and

$$A_m(z^*) \subseteq A_{m-1}.$$

So  $A_m = \bigcup_{z \in A_{m-1}} A_m(z) \subseteq A_{m-1}$ .

LEMMA 2: The Steady State set,  $M$ , is a subset of any Attainable set,  $A_m$ .



Proof:  $M = \{x; x \geq x \cdot P, x \in F\}$ .  $M \subseteq A_0 = F$  by definition.

Assume  $M \subseteq A_{m-1}$ . Pick  $z^* \in M \Rightarrow z^* \in A_{m-1}$  and  $z^* \geq z^* \cdot P$ .

Therefore,  $z^* \in A_m$  by definition of  $A_m$ , and  $M \subseteq A_m$ .

LEMMA 3: Any Reachable set,  $B_m$ , contains every previously reachable set.

Proof:  $M = B_0 = \{x; x \geq z \cdot P, x \in F\}$ . Trivially,  $B_0 \subseteq B_1$ .

Assume that  $B_{m-2} \subseteq B_{m-1}$  where  $B_{m-1} = \{x; x \geq z \cdot P, x \in F, z \in B_{m-2}\}$ .

Define the following sets:  $B_{m-1}(z) \equiv \{x; x \geq z \cdot P, x \in F\}$ ,  $z \in B_{m-2}$ , and  $B_m(z) \equiv \{x; x \geq z \cdot P, x \in F\}$ ,  $z \in B_{m-1}$ . Pick  $z^* \in B_{m-2}$ . Since  $B_{m-2} \subseteq B_{m-1}$ ,  $z^* \in B_{m-1}$ , so that  $B_{m-1}(z^*) = B_m(z^*)$ . Since  $z^*$  is arbitrary, and since  $B_{m-2} \subseteq B_{m-1}$ ,  $B_{m-1} = \bigcup_{z \in B_{m-2}} B_{m-1}(z) \subseteq \bigcup_{z \in B_{m-1}} B_m(z) = B_m$ .

LEMMA 4: If  $w_i > 0$  for at least one  $i$  and  $n > 1$ , then the Steady State set,  $M$ , contains an infinite number of points.

Proof:

(i) Non-empty. Assume that  $M$  is empty. Then there does not exist an  $x$  such that  $x \cdot (I-P) \geq 0$ . Because

$$\sum_{j=1}^n p_{ij} \leq 1, i=1,2,\dots,n \text{ with strict inequality holding}$$

for at least one  $i$ ,  $(I-P)^{-1}$  exists with all elements non-negative. Let  $N = (I-P)^{-1}$ . If  $M$  is empty, then there does not exist an  $x$  such that  $x = f \cdot N \geq 0$ . But

$$x_j = \sum_{i=1}^n f_i \cdot n_{ij} \text{ and since } f_i \geq 0 \text{ and } n_{ij} \geq 0 \text{ for every}$$

$i, j$ ,  $x_j \geq 0$  and therefore  $M$  is non-empty.



(ii) Infinite Interior. Assume that  $M$  contains only one point, then  $x(t + 1) = x(t)$  for every  $t$  and any appointment vector. We rewrite equation (1) as  $x(t) = x(t) \cdot P + f(t + 1) = f(t + 1) \cdot (I - P)^{-1}$ . We also require that  $f(t + 1) \cdot e = x(t) \cdot w$ . We can construct the appointment vector in the following manner: let  $f(t + 1) = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n$  where  $\sum_{i=1}^n \alpha_i = x(t) \cdot w$ . Then  $f(t + 1) = (\alpha_1, \alpha_2, \dots, \alpha_n)$  and

$$x(t) = (\alpha_1, \alpha_2, \dots, \alpha_n) \cdot (I - P)^{-1} \quad (3)$$

Consider the appointment vector  $f(t + 1)$  when  $\alpha_1 = x(t) \cdot w$  and  $\alpha_2 = \alpha_3 = \dots = \alpha_n = 0$ , then  $x(t)$  equals  $x(t) \cdot w$  multiplying the first row of  $(I - P)^{-1}$ . Since any  $f$  which solves (3) generates a point in  $M$ , consider the appointment vector  $f(t + 1)$  when  $\alpha_2 = x(t) \cdot w$  and  $\alpha_1 = \alpha_3 = \dots = \alpha_n = 0$ , then  $x(t)$  equals  $x(t) \cdot w$  multiplying the second row of  $(I - P)^{-1}$ . Since  $x(t)$  is one point, and  $x(t) \cdot w$  is a scalar, the first two rows of  $(I - P)^{-1}$  are identical. This implies that  $r((I - P)^{-1}) < n$  and that  $((I - P)^{-1})^{-1}$  does not exist which is a contradiction. Thus since  $M$  is non-empty,  $M$  contains more than one point. Since  $M$  is convex and closed, it contains an infinite number of points.

**THEOREM 1:** The Containment Set,  $C$ , and the Reached Set,  $B$ , exist, are identical, and non-empty.



Proof: Consider the sequence of sets  $\{A_m\}$ . By Lemmas 1 and 2,  $M \subseteq \dots \subseteq A_m \subseteq A_{m-1} \subseteq \dots \subseteq A_1 \subseteq F$ . By Lemma 4  $M \neq \emptyset \Rightarrow A_m \neq \emptyset, n = 1, 2, \dots$ . Thus we have a monotone decreasing sequence of sets which is bounded below, therefore the limiting set does exist and is non-empty. Now consider the sequence of sets  $\{B_m\}$  where  $B_0 = M$ . By Lemma 3,  $M \subseteq B_0 \subseteq B_1 \subseteq \dots \subseteq B_{m-1} \subseteq B_m \subseteq \dots \subseteq F$ . Thus we have a monotone increasing sequence of sets bounded above by  $F$ . Therefore a limiting set exists. As noted in section III the Containment set,  $C$ , and the Reached set,  $B$ , are the same set.

THEOREM 2: The Steady State set,  $M$ , any Attainable set,  $A_m$ , the Containment set,  $C$ , any Reachable set,  $B_m$ , and the set of all rank distributions,  $F$ , are identical if and only if the Promotion matrix,  $P$ , is diagonal.

Proof: Since  $M \subseteq B_m \subseteq C \subseteq A_m \subseteq F$ , we need only show that  $M = F$ .

(i) Assume that  $P$  is diagonal. We can write the distribution vector as  $x(t) = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n$ , where  $\alpha_i \geq 0$ . If  $\sum \alpha_i = 1$  then  $x(t) \in F$ . Rewrite equation (1)  $x(t+1) = x(t) \cdot P + f(t+1)$ . We require  $f(t+1) \cdot e = x(t) \cdot (I-P) \cdot e$ . Since  $P$  is diagonal,  $x(t) \cdot (I-P)$  is a suitable choice for  $f(t+1)$ , ( $f_i(t+1) \geq 0$ ). So we can now write  $x(t+1) = x(t) \cdot P + x(t) \cdot (I-P) = x(t)$ . Now  $x(t+1) = x(t)$  and  $x(t) \in F \Rightarrow x(t) \in M, \therefore F \subseteq M$ . And since  $M \subseteq F$ ,  $M = F$ .



(ii) Assume  $M = F$ .  $x(t + 1) = x(t)$  for all  $t$ .

Assume that  $P$  is not diagonal.  $x(t) = x(t) \cdot P + f(t + 1)$ .

since  $x(t) \in F$ , we can write  $x(t) = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n$ ,  $\alpha_i \geq 0$ ,  $\sum_i \alpha_i = 1$ . In particular, let  $\alpha_1 = 1$ ,

$\alpha_i = 0$ ,  $i \neq 1$ . So  $x(t) = \alpha_1 e_1$  and  $(1, 0, \dots, 0) =$

$$(1, 0, \dots, 0) \cdot \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ \vdots & & & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix} + f(t + 1)$$

$= (p_{11}, p_{12}, \dots, p_{1n}) + (f_1, f_2, \dots, f_n)$ . This requires that the following equations be satisfied:

$$\begin{array}{rcl} f_1 + p_{11} & = & 1 \\ f_2 + p_{12} & = & 0 \\ \vdots & & \vdots \\ f_n + p_{1n} & = & 0 \end{array} .$$

Since  $f_j \geq 0$ ,  $p_{ij} \geq 0$ , we have  $f_j = p_{1j} = 0$ ,  $j \neq 1$ . We repeat this procedure for  $\alpha_j = 1$ ,  $\alpha_k = 0$ ,  $k \neq j$ ,  $j = 2, 3, \dots, n$ . Thus  $p_{ij} = 0$ ,  $i \neq j$  and  $P$  is diagonal.



## V. EXTREME POINTS OF THE VARIOUS SETS

The characterizations of the various sets in the model in the above section do not describe them in a useful manner. If we knew the extreme points of all of the sets, then we would have almost all of the information about the sets, since any interior point of a set is simply the convex combination of its extreme points.

For two sets in particular, the extreme points are easily obtained. The set of all rank distributions,  $F$ , is the fundamental simplex in  $E_n$  with extreme points,  $e_i$ ,  $i = 1, 2, \dots, n$ . The subset of  $F$  which is most easily described is the Steady State set,  $M$ . Each vector in the set satisfies the inequality  $x \geq x \cdot P$ . This can be written in equation form as  $x \cdot (I - P) = f$  where  $f$  is the vector of appointments to the different ranks. We define  $N = (I - P)^{-1}$  so we write  $x = f \cdot N$ . Since we require  $x \cdot e = 1$  and since  $f$  can be written  $f = \sum_i \alpha_i (x \cdot (I - P) \cdot e)$ ,  $\alpha_i \geq 0$ ,  $\sum_i \alpha_i = 1$ , we write the equation in an equivalent form as  $x = f \cdot B$

where  $f = \sum_i \alpha_i e_i$  and  $B_{ij} = \frac{N_{ij}}{\sum_j N_{ij}}$ . That is, the extreme points of

the Steady State set are given by the rows of  $(I - P)^{-1}$  after the rows are normalized.

The problem of determining the extreme points of the other sets in the model is much more difficult. If we are working in three dimensions, it is not too difficult to calculate all of the extreme points of one of the sets,  $A_m$  or  $B_m$ , for a small  $m$ .



Each set in the model has the property that each point in the set generates a subsimplex in the fundamental simplex. The extreme points of the generated set are found from the convex hull of the subsimplices generated by the extreme points of the original set. In three dimensions we use a geometrical method to determine the extreme points of the convex hull of the generated subsimplices. If we know the extreme points of  $A_m$  for some  $m$ , then we project  $A_m$  onto a plane generated by two unit vectors. Then we calculate the extreme points of the subsimplex generated by each extreme point of  $A_m$  and project these onto the plane. Then we simply test linear combinations of the generated extreme points to determine that there are no points which lie beyond the linear combination in each of the unit directions. The points which satisfy this criterion are the extreme points of the set  $A_{m+1}$ . A similar procedure is used for  $B_m$ .

When  $P$  is upper triangular, some of the extreme points of  $A_m$  are also extreme points of  $A_{m+1}$  (e.g.  $e_n$  is an extreme point of  $A_m$  for every  $m$ .) These remain extreme points for all succeeding sets generated from  $A_{m+1}$ . Similarly,  $B_m$  has extreme points common to all  $B_m$  which are extreme points of  $A_m$  as well, and therefore are extreme points of the Containment set,  $C$ . Figure 1 depicts several attainable sets drawn on barycentric coordinates for a three dimensional rank distribution where



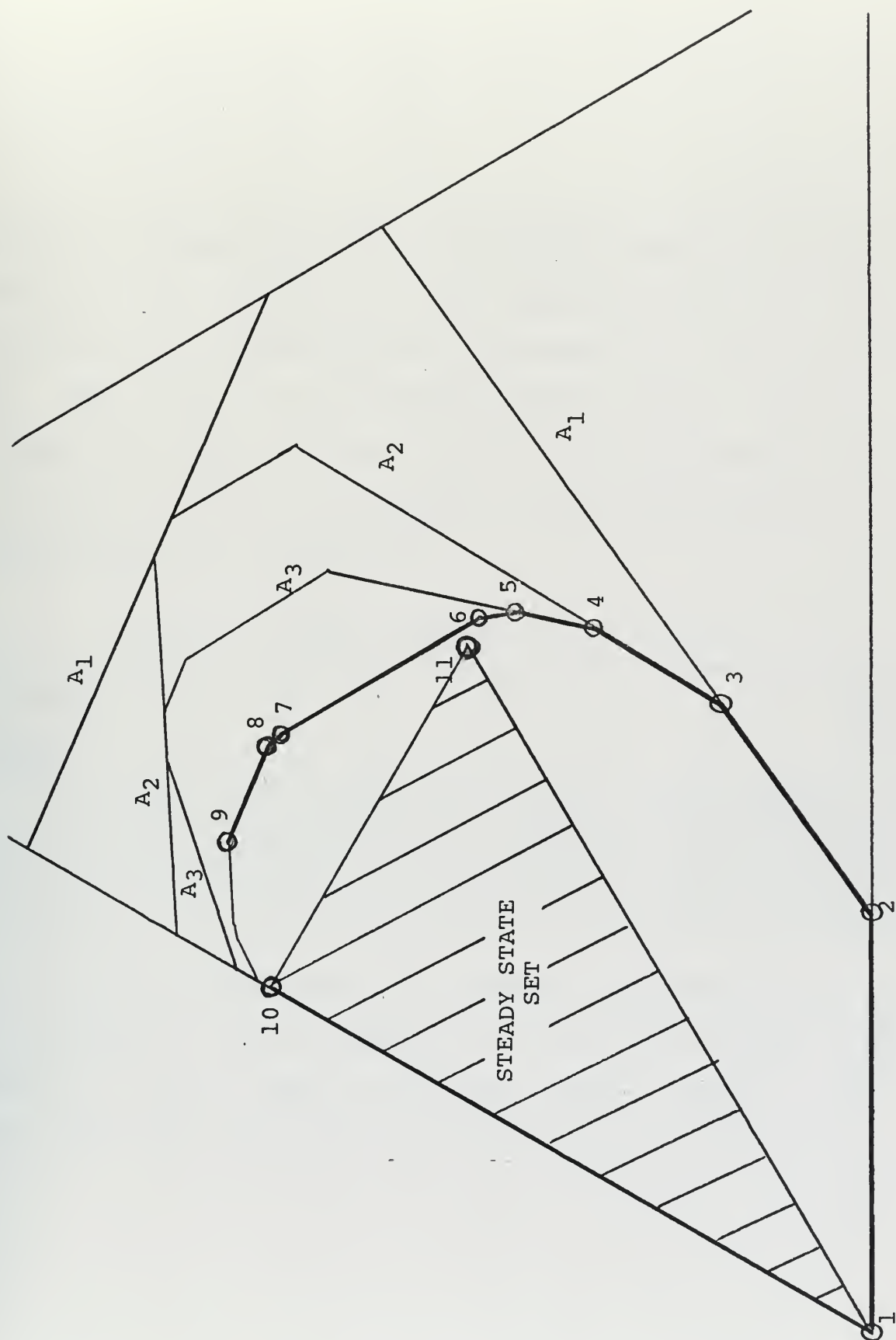


FIGURE 1. ATTAINABLE SETS ON BARYCENTRIC COORDINATES



$$P = \begin{bmatrix} .5 & .4 & 0 \\ 0 & .6 & .3 \\ 0 & 0 & .7 \end{bmatrix} .$$

Table I contains the coordinates of the numbered points on Figure 1. The heavy line in the figure represents the known portion of the boundary of the Containment set after calculating  $A_7$ . The portion of the boundary between points 9 and 10 changed very little between  $A_6$  and  $A_7$  so the boundary of  $A_7$  is probably a good approximation of the boundary of the Containment set in this case.

TABLE I: POINTS ASSOCIATED WITH FIGURE 1.

Point	1	2	3	4	5	6	7	8	9	10	11
Rank											
1	0	.3	.39	.393	.3723	.35289	.18615	.176445	.0882225	0	1/3
2	0	0	.12	.228	.2940	.32532	.49206	.500706	.5355714	.5	1/3
3	1	.7	.49	.379	.3337	.32179	.32179	.322849	.3762061	.5	1/3

It is not clear how this graphical projection method can be extended to higher dimensions to obtain the extreme points. Nor is it clear, based on the above example, whether or not the Containment set has a finite number of extreme points.



## VI. THE DEVELOPMENT OF SEVERAL TESTS

Since it is not feasible to analytically determine the extreme points of all of the sets, we must devise some method to answer our original questions. The answers to those questions depend on knowing the extreme points of all of the sets. To provide partial answers to the questions, we develop test procedures to determine if a given vector is an element of a particular set, and to determine if a given vector can be returned to in  $m$  steps or reached from another specified vector in  $m$  steps. We know that a given vector cannot be returned to at any time if there exists an  $i$  such that the test vector is not in  $A_i$ .

Each transition from one set to another requires that a certain set of inequalities be satisfied. When we consider the system of inequalities that account for all of the transitions, we need only test for feasibility. If no feasible solution exists, then the test vector fails the test. Several specific tests are developed in the next sections.

### A. A TEST FOR $B_m$

In this section, we develop a test to determine if a given rank distribution is an element of the set  $B_m$ . Recall that  $B_m = \{z_m; z_m \geq z_{m-1} \cdot P, z_m \in F, z_{m-1} \in B_{m-1}\}$  for  $m = 1, 2, \dots$  and that  $B_m$  is the set of rank distributions which is reachable in  $m$  steps from the Steady State set. Row vectors have been used in all developments. To simplify notation, the test inequalities will be specified in tableau form and vectors



will be column vectors. Transposition is indicated for the promotion matrix where it appears.

We now write the sequence of inequalities which must be satisfied if the vector  $X$  is in the set  $B_m$  as

$$\begin{array}{ccccccccccc}
 z_0 & & z_1 & & z_2 & & \cdot & & \cdot & & z_{m-2} & & z_{m-1} \\
 (P^T - I) & & & & & & & & & & & & & \leq 0 \\
 P^T & & -I & & & & & & & & & & & \leq 0 \\
 & & P^T & & -I & & & & & & & & & \leq 0 \\
 & & & & \cdot & & & & & & & & & \cdot \\
 & & & & & & \cdot & & & & & & & \cdot \\
 & & & & & & & & P^T & & -I & & & \leq 0 \\
 & & & & & & & & & & P^T & & & \leq X^T \quad (4) \\
 e & & & & & & & & & & & & & = 1 \\
 & & e & & & & & & & & & & & = 1 \\
 & & & & \cdot & & & & & & & & & \cdot \\
 & & & & & & \cdot & & & & & & & \cdot \\
 & & & & & & & & e & & & & & = 1 \\
 & & & & & & & & & & e & & & = 1
 \end{array}$$

The constraint matrix appears deceptively smaller than it is. Since each  $z_i$  is an  $n$  vector, the dimensions of the matrix are  $mn + m + n$  rows and  $mn$  columns. Even though the constraint matrix is very sparse, we were unable to apply any algorithm which would process the problem more efficiently than the Simplex algorithm.



It is clear that a feasible solution to (4) is simply a Phase I Simplex solution. The method which we use to obtain this solution is to add an artificial variable to each of the feasibility constraints, the last  $m$  rows. We then minimize the sum of the artificial variables. A feasible solution to the new system, (4) plus the artificial variables, always exists. The result of the test is determined by the values of the artificial variables. If all of the artificial variables are equal to zero, then (4) has a feasible solution and we conclude that the test vector,  $X$ , is an element of  $B_m$ . If any of the artificial variables are not zero, then a feasible solution to (4) does not exist and we conclude that  $X$  is not in  $B_m$ .

#### B. A TEST FOR $A_m$

In this section, we develop a test to determine if a given rank distribution is an element of  $A_m$ , i.e. if it can be reached in  $m$  steps from some point in  $F$ . Recall that

$$A_m = \{z_m; z_m \geq z_{m-1} \cdot P, z_m \in F, z_{m-1} \in A_{m-1}\} \text{ for } m = 1, 2, \dots$$

We write the set of inequalities as



$$\begin{array}{cccccccc}
z_0 & z_1 & z_2 & \cdot & \cdot & \cdot & z_{m-2} & z_{m-1} \\
p^T & -I & & & & & & \leq 0 \\
& p^T & -I & & & & & \leq 0 \\
& & \cdot & & & & & \cdot \\
& & & \cdot & & & & \cdot \\
& & & & \cdot & & & \cdot \\
& & & & & p^T & -I & \leq 0 \\
& & & & & & p^T & \leq x^T \quad (5) \\
e & & & & & & & = 1 \\
& e & & & & & & = 1 \\
& & \cdot & & & & & \vdots \\
& & & \cdot & & & & \vdots \\
& & & & \cdot & & & \vdots \\
& & & & & e & & = 1 \\
& & & & & & e & = 1
\end{array}$$

The constraint matrix has  $mn + m$  rows and  $mn$  columns. We now test (5) for a feasible solution in the same manner as the test for  $B_m$ . The results of the test are interpreted in the same manner as those for  $B_m$ . Note that the constraint matrix in (5) is the same as that in (4) without the first  $n$  rows.

### C. A TEST FOR RETURN IN $m$ STEPS

In this section, we develop a test to determine whether or not a given rank distribution,  $X$ , can be returned to in  $m$  steps. If  $X$  is an element of the Steady State set, we know that it can be returned to in one step, i.e. it can be maintained forever. So we are interested in a test for  $m$



greater than one. Since there are  $m$  transitions, there must be  $m-1$  intermediate vectors because the initial and terminal vectors are both  $X$ . The first intermediate vector,  $z_1$ , must satisfy  $z_1 \geq X \cdot P$ . The second intermediate vector,  $z_2$ , must satisfy  $z_2 \geq z_1 \cdot P$ . This process continues until the last intermediate vector,  $z_{m-1}$ , must satisfy  $z_{m-1} \cdot P \leq X$ . We write the sequence of inequalities in tableau form as

$$\begin{array}{ccccccccc}
 z_1 & z_2 & & \cdot & \cdot & \cdot & z_{m-2} & z_{m-1} & \\
 I & & & & & & & & \geq P^T \cdot X^T \\
 P^T & -I & & & & & & & \leq 0 \\
 & P^T & & & & & & & \leq 0 \\
 & & \cdot & & & & & & \cdot \\
 & & & \cdot & & & & & \cdot \\
 & & & & \cdot & & & & \cdot \\
 & & & & & P^T & -I & & \leq 0 \\
 & & & & & & P^T & & \leq X^T \\
 e & & & & & & & & = 1 \\
 & e & & & & & & & = 1 \\
 & & \cdot & & & & & & \vdots \\
 & & & \cdot & & & & & \\
 & & & & \cdot & & & & \\
 & & & & & e & & & = 1 \\
 & & & & & & & & \\
 & & & & & & e & & = 1
 \end{array} \tag{6}$$

Again we notice that there is a great similarity between the form of this tableau and those for the tests for  $A_m$  and



$B_m$ . The first  $n$  inequalities in (6) are different from those in (4), as well as there being one less intermediate vector. The constraint matrix in (6) has  $mn + m + n$  rows and  $mn - n$  columns.

As with the tests for  $A_m$  and  $B_m$ , any method of finding a feasible solution to (6) is acceptable. One method would be to add artificial variables not only to the last  $m$  rows of (6), but also to the first  $n$  rows, then minimizing the sum of the artificial variables. If all of the artificial variables are zero, then we conclude that the rank distribution,  $X$ , can be returned to in  $m$  steps. The method used in section VII involved adding artificial variables only to the last  $m$  rows of (6), then minimizing the sum of the artificial variables. This system may not always have a feasible solution. The test requires that if the system has a feasible solution, then if all of the artificial variables are zero, we conclude that  $X$  can be returned to in  $m$  steps.

#### D. A TEST FOR CHANGING DISTRIBUTIONS IN $m$ STEPS

We now develop a test to determine, given some distribution vector,  $X_1$ , whether or not we can reach another specified distribution vector,  $X_2$ , in  $m$  steps. This is nearly identical to the problem discussed in the above section, i.e. we have a fixed initial vector and a fixed terminal vector. The constraint matrix is the same as (6) but now the right hand side is  $[P^T \cdot X_1^T, 0, \dots, 0, X_2^T, 1, \dots, 1]$ . The test procedure is then the same as the test for return in  $m$  steps.



# E. A TEST FOR EXTREME POINTS OF THE CONTAINMENT SET

The similarity between the tests for  $A_m$  and  $B_m$  noted in sections A and B above permits us to develop a procedure with which we can determine extreme points of the Containment set, C.

Let  $G_1 = [(I-P^T), 0, \dots, 0]$  be an  $n$  by  $mn$  matrix and let

$$G_2 = \begin{bmatrix} P^T & -I & & & & & & & \\ & P^T & -I & & & & & & \\ & & \cdot & & & & & & \\ & & & \cdot & & & & & \\ & & & & \cdot & & & & \\ & & & & & P^T & -I & & \\ & & & & & & P^T & & \\ & e & & & & & & & \\ -e & & & & & & & & \\ & e & & & & & & & \\ -e & & & & & & & & \\ & & \cdot & & & & & & \\ & & & \cdot & & & & & \\ & & & & e & & & & \\ & & & & -e & & & & \\ & & & & & e & & & \\ & & & & & -e & & & \\ & & & & & & e & & \\ & & & & & & -e & & \end{bmatrix}$$

be an  $mn + 2m$  by  $mn$  matrix. Now let  $X_2 = [0, \dots, 0, x^T, 1, \dots, 1]$  be an  $mn + 2m$  by 1 column vector and  $Z = [z_0^T, \dots, z_{m-1}^T]$  be an  $mn$  by 1 column vector. Then the test for  $A_m$  reduces to finding



a feasible solution to

$$G_2 \cdot Z \leq X_2 . \quad (7)$$

If there is a  $Z^*$  which satisfies (7) then the test for  $B_m$  would seem to result in testing to see if  $G_1 \cdot Z^* \geq 0$ . However, there is no guarantee that  $Z^*$  is unique and there could exist a  $Z'$  which solves (7) and also solves  $G_1 \cdot Z' \geq 0$  for the same test vector  $X$ . So this would be a sufficient test for inclusion in  $B_m$ , but not necessary. The only necessary and sufficient test is that described in section III.A.

Now let  $H_1 = [(I-P^T), 0, \dots, 0]$  be an  $n$  by  $mn + n$  matrix and let

$$H_2 = \begin{bmatrix} p^T & -I & & & & & & & \\ & p^T & -I & & & & & & \\ & & & \ddots & & & & & \\ & & & & -I & & & & \\ & & & & & p^T & -I & & \\ & e & & & & & & & \\ & -e & & & & & & & \\ & & e & & & & & & \\ & & -e & & & & & & \\ & & & & & e & & & \\ & & & & & -e & & & \\ & & & & & & e & & \\ & & & & & & -e & & \end{bmatrix}$$



be an  $mn + n + 2m$  by  $mn + n$  matrix. Let  $Z' = [z_0^T, \dots, z_m^T]$  be an  $mn + n$  by 1 column vector and let  $X_2' = [0, \dots, 0, 1, \dots, 1]$  be an  $mn + n + 2m$  by 1 column vector. This amounts to letting the test vector,  $x_i^T$ , become an unknown,  $z_m^T$ . Then all of the basic feasible solutions to  $H_2 \cdot Z' \leq X_2'$  are extreme points of  $A_m$ . If a point is an extreme point of  $A_m$  and  $B_m$  simultaneously, then it is an extreme point of the Containment set,  $C$ . Thus we need only test each basic feasible solution to see if  $H_1 \cdot Z' \geq 0$ . If it is, then  $Z'$  is an extreme point of  $C$ .

Due to the computational difficulties associated with this test, it is presented for theoretical interest only.



## VII. A CASE STUDY

The tests developed in section VI above were applied to faculty distributions in the College of Engineering at the University of California at Berkeley based on data from 1960 to 1968. The bulk of the data is in Branchflower [1970] and only that which is relevant is reproduced here.

We used the Mathematical Programming System on the IBM 360/67 computer at the W. R. CHURCH Computer Center, Naval Postgraduate School to conduct most of the tests. This is really a trivial exercise once the data cards are prepared for the control programs. One FORTRAN IV program was written to generate the data cards for the test for  $B_m$ . The same cards were used to test for  $A_m$  after removing the cards pertaining to the first  $n$  rows of the constraint matrix. Another FORTRAN IV program was written which was used to generate data cards for testing return in  $m$  steps. Again, hand modification of the right hand side permitted the same cards to be used to test for changing distributions in  $m$  steps.

One other FORTRAN IV program was written which was used to determine the extreme points of the steady state set. It was also used to test to determine if a given vector was an element of the Steady State set.

All of the tests were verified for accuracy by using a test case, namely the three state promotion matrix in section V and points which were in known sets from Figure 1.



STATE

I \ J	1	2	3	4	5	6	7	8	9	10	11	12	13
1	.28	.56	.04	.04									
2		.41	.47	.07									
3			.44	.37	.14								
4				.38	.52	.04							
5					.47	.43	.10						
6						.57	.41	.01					
7							.64	.34					
8								.67	.30	.01			
9									.69	.25	.02		
10										.71	.24	.01	
11											.73	.25	.01
12												.77	.20
13													.97

FIGURE 2. PROMOTION MATRIX FROM 1960-1968 DATA

TABLE II. TEST VECTORS

VECTOR STATE	X <sub>1</sub> (1965)	X <sub>2</sub> (1966)	X <sub>3</sub> (1967)	X <sub>4</sub> (1968)
1	.0161427	.0154639	.0	.0047619
2	.0802139	.0567010	.0628019	.0190476
3	.0534759	.0979381	.0821256	.0809523
4	.0374331	.0257731	.0628019	.0761904
5	.0534759	.0618556	.0483091	.0523809
6	.0481283	.0463917	.0628019	.0666666
7	.1390374	.0979381	.0821256	.0952380
8	.1176470	.1494845	.1400966	.1047619
9	.1122994	.1134020	.1159420	.1238095
10	.1390374	.1030927	.1062801	.0904761
11	.0962566	.1185567	.1014492	.1047619
12	.0855614	.0670103	.0676328	.1047619
13	.0213903	.0463917	.0717328	.0761904



For the case study, we used the thirteen active states of the faculty distribution and the 1960-1968 promotion matrix from Branchflower [1970]. This is reproduced as Figure 2. The actual distributions of faculty for 1965-1968 were normalized and used as the four test vectors. These are included in Table II.

The test vectors were tested for being elements of  $A_m$  and  $B_m$ ,  $m = 1, 2, \dots, 7$ , and were tested to determine if they could be returned to in  $m$  steps for  $m = 2, 3, \dots, 8$ . The resulting location of the vectors of rank distributions on the fundamental simplex is indicated in Table III.

TABLE III. TEST RESULTS

$X_1$	$X_2$	$X_3$	$X_4$
$X_1 \in A_2$		$X_3 \in A_2$	$X_4 \in A_2$
$X_1 \notin A_3$	$X_2 \notin A_1$	$X_3 \notin A_3$	$X_4 \notin A_3$

Recall that if  $X_i \in A_j$  then after one step,  $X_{i+1} \geq X_i \cdot P$ ,  $X_{i+1}$  is at least an element of  $A_{j+1}$ . The results in Table III are inconsistent with this theoretical result and require an explanation. There are two reasons why this could have occurred. The first reason is that the Promotion matrix was an average over an eight year period and did not exactly represent the true Promotion matrix for each year. The second reason is that the faculty size did not remain constant, but in fact was increasing. An expanding organization can appoint personnel



to replace the losses and then hire the additional number for expansion into any state. When  $P$  is upper triangular, the extreme points of successive  $A_m$  may tend to move away from the lower ranks. For a three state example, consider an initial rank distribution of  $(1, 0, 0)$  with

$$P = \begin{bmatrix} .5 & .4 & 0 \\ 0 & .6 & .3 \\ 0 & 0 & .7 \end{bmatrix}$$

then after one transition, the rank distribution before new appointments is  $(.5, .4, 0)$ . Say the number of persons in the organization is constant at 100, then the distribution of persons in the ranks is  $(50, 40, 0)$  with a loss of 10 to replace. Appointments could result in the distributions  $(60, 40, 0)$ ,  $(50, 50, 0)$ , or  $(50, 40, 10)$  or any linear combination of them. If the organization expands by 10 persons, then the distributions could be  $(70, 40, 0)$ ,  $(50, 60, 0)$ , or  $(50, 40, 20)$  or any linear combination of them. If we normalize, then these expanded distributions are outside  $A_1$  for the constant size case. The result is that the possible distributions for the expanding size model can be outside of the normal set of distributions obtained for the constant size model.

We then aggregated several ranks in the model to determine if the aggregated model behaved in the same way. The first four ranks are Assistant Professors, the next three ranks are



Associate Professors, and the last six ranks are Full Professors. The resulting Promotion matrix is

$$P = \begin{bmatrix} .7649 & .1752 & 0 \\ 0 & .8030 & .1841 \\ 0 & 0 & .9707 \end{bmatrix}$$

The normalized rank distributions for 1965-1968 are listed in Table IV.

TABLE IV. TEST VECTORS: AGGREGATED MODEL

VECTOR RANK	$X_1$ (1965)	$X_2$ (1966)	$X_3$ (1967)	$X_4$ (1968)
Asst. Prof.	.187166	.195876	.207729	.180952
Assoc. Prof.	.240642	.206186	.193237	.214286
Full Prof.	.572192	.597938	.599034	.604762

These vectors were tested to determine their location on the fundamental simplex. The results are shown in Table V.

TABLE V. TEST RESULTS: AGGREGATED MODEL

$X_1$	$X_2$	$X_3$	$X_4$
$X_1 \in A_9$	$X_2 \in A_7$	$X_3 \in A_6$	$X_4 \in A_9$
$X_1 \notin A_{10}$	$X_2 \notin A_8$	$X_3 \notin A_7$	$X_4 \notin A_{10}$



These results using the aggregated model are similar to the results of the thirteen state model. We noted above that as faculty size increases, the rank distribution may move to a superset of the set containing the original distribution. In fact, this happens in 1966 for an increase of seven faculty members (3.7%), and also in 1967 for an increase of thirteen faculty members. In 1968, there was an increase of three faculty members, but the rank distribution moved to a subset of the set containing the 1967 rank distribution.



### VIII. CONCLUSIONS AND RECOMMENDATIONS

We have established several properties of several sets of rank distribution in an hierarchical organization which have not been previously examined. In addition, based on these properties, we developed several tests to determine if a specified rank distribution is an element of certain sets. The tests were applied to data from a case study of the College of Engineering faculty at the University of California, Berkeley.

Several questions remain unanswered and are possible areas for further study. Is there some neighborhood of a given rank distribution which can be maintained? Is there an efficient algorithm which will solve the test inequalities other than the Simplex method? Is there a method of reducing the number of basic feasible solutions which must be found to determine the extreme points of the Containment set? Does the Containment set have a finite number of extreme points?



```

* * * * *
* * * * DATA GENERATOR - TEST FOR RETURN IN M STEPS * * * *
* * * * * ROBERT L. ARMACOST - - - 1 JULY 1970 * * * * *
* * * * *
* * * * *

```

```

THIS PROGRAM WILL PUNCH OUT A SET OF DATA CARDS FOR A
LINEAR PROGRAM USING THE MPS PACKAGE ON THE IBM 360/67.
THE BASIC INPUT TO THE PROGRAM IS A SQUARE PROMOTION
MATRIX OF SIZE N, AND K VECTORS WHICH ARE TO BE TESTED
TO DETERMINE IF THE VECTOR CAN BE RETURNED TO IN M
STEPS. THE PROGRAM WILL GENERATE DATA DECKS FOR A
SERIES OF TESTS TO RETURN IN M STEPS. MSTART IS THE
STARTING INDEX FOR THE NUMBER OF STEPS TO RETURN, AND
MSTOP IS THE ENDING INDEX. THESE INDICES ARE READ IN
AS DATA. THE FIRST VECTOR TO BE TESTED IS CALLED
"TEST". EACH SUCCEEDING VECTOR IS CALLED "REVI" WHERE
I IS THE NUMBER OF THE REVISION OF THE RIGHT HAND SIDE.
THE VARIABLE IR IS THE NUMBER OF REVISIONS IN THE
CONTROL PROGRAM. THE ARRAYS SHOULD BE DIMENSIONED IN
THE FOLLOWING WAY -
    AX((MSTOP-1)*(N+1) + N, (MSTOP-1)*(N+1))
    KEY - SAME AS AX
    X(N,IR+1)
    P(N,N)
    PT(N,N)
    PX(N,N)
    UNI(N,N).

```

```

THE PROGRAM WILL ACCOMODATE A PROMOTION MATRIX WITH
N = 99 AND WITH MSTOP < 100.

```

```

* * * * *
* * * * *

```

```

THERE IS A CERTAIN AMOUNT OF REPETITION BETWEEN THIS
PROGRAM AND THE FOLLOWING PROGRAM FOR A TEST FOR A
REACHABLE SET IN M STEPS. TO MINIMIZE DOCUMENTATION,
CERTAIN MODULES IN THIS PROGRAM NEED ONLY BE REPLACED
BY OTHER MODULES TO OBTAIN THE NEW PROGRAM. THESE
MODULES ARE MARKED WITH A NUMBER IN THE LEFT HAND
COLUMN WHICH MEANS THEY ARE TO BE REPLACED BY THE
STATEMENTS WITH THE CORRESPONDING NUMBER IN THE NEXT
SECTION TO OBTAIN THE NEW PROGRAM. IN ADDITION, THERE
ARE SEVERAL STATEMENTS WHICH NEED ONLY BE REMOVED.
THESE ARE MARKED WITH AN R IN THE LEFT HAND COLUMN.

```

```

* * * * *

```

```

INTEGER*2 KEY(111,98)
DIMENSION AX(111,98), X(13,4), P(13,13), PT(13,13),
1UNI(13,13), PX(13,13)

```

```

READ INPUT DATA

```

```

1 READ(5,600) MSTART, MSTOP, N, IR
600 FORMAT(4I5)

```



```

        IR1 = IR + 1
        READ(5,601) ((P(I,J),J=1,N),I=1,N)
601    FORMAT(8F10.0)
        READ(5,601) ((X(I,J),I=1,N),J=1,IR1)
R      MBEG = MSTART - 1
R      MEND = MSTOP - 1
        DO 999 M = MBEG,MEND

        INITIALIZE

        ITEST = 1
        JLIM = M*N + M
        ILIM = JLIM + N
        DO 10 J=1,JLIM
        DO 10 I=1,ILIM
        AX(I,J) = 0.
10     KEY(I,J) = 0

2      CALCULATE THE TRANSPOSE OF THE PROMOTION MATRIX, PT,
2      AND LOAD THE IDENTITY MATRIX UNI
2
2      DO 20 I=1,N
2      DO 20 J=1,N
2      UNI(I,J) = 0.
2      PT(I,J) = P(J,I)
2      IF(I.EQ.J) UNI(I,J) = 1.
2 20 CONTINUE

R      CALCULATE THE RHS FOR THE FIRST N ROWS
R
R      DO 1 J = 1,IR1
R      DO 2 I = 1,N
R      SUM = 0.
R      DO 3 K = 1,N
R 3 SUM = SUM + PT(I,K) * X(K,J)
R 2 PX(I,J) = SUM
R 1 CONTINUE

3      INSERT UNI IN THE FIRST N ROWS AND COLUMNS OF THE
3      CONSTRAINT MATRIX, AX
3
3      DO 30 I=1,N
3      DO 30 J=1,N
3      AX(I,J) = UNI(I,J)
3 30 IF(AX(I,J).NE.0) KEY(I,J) = 1

4      INSERT PT AND -I IN THE APPROPRIATE COLUMNS OF THE NEXT
4      (M-2)*N ROWS OF AX

        LASTC = N
        NXTR = N+1
        NXTC = N+1
        JLASTC = 1
        LASTR = 2*N
        JNXTC = 2*N
        MOD = 1
5      IF(MOD.EQ.M) GO TO 100
        DO 50 I = NXTR,LASTR
        DO 40 J = JLASTC,LASTC
        AX(I,J) = PT(I-MOD*N,J-(MOD-1)*N)
40     IF(AX(I,J).NE.0) KEY(I,J) = 1
        DO 50 J = NXTC,JNXTC
        IF(I.EQ.J) AX(I,J) = -1.0
50     IF(AX(I,J).EQ.-1.0) KEY(I,J) = 1
        NXTC = NXTC + N
        NXTR = NXTR + N
        LASTC = JNXTC
        JLASTC = NXTC - N
        LASTR = NXTR + N - 1
        JNXTC = LASTC + N
        MOD = MOD + 1
        GO TO 5

```



INSERT PT IN THE CORRECT COLUMNS OF THE NEXT N ROWS OF  
AX

```
100 DO 60 I = NXTR, LASTR
    DO 60 J = JLASTC, LASTC
    AX(I,J) = PT(I-MOD*N, J-(MOD-1)*N)
60 IF(AX(I,J).NE.0) KEY(I,J) = 1
```

INSERT 1'S IN THE CORRECT COLUMNS OF THE LAST M ROWS OF  
AX WHICH CORRESPOND TO THE FEASIBILITY CONSTRAINT

```
MOD = MOD + 1
IM = LASTR + 1
IMM = LASTR + M
DO 70 I=IM, IMM
ICOL = (I-IM)*N + 1
JCOL = ICOL - 1 + N
DO 65 J = ICOL, JCOL
AX(I,J) = 1
65 KEY(I,J) = 1
K = M*N + I - LASTR
AX(I,K) = 1
KEY(I,K) = 1
70 CONTINUE
```

PUNCH OUT THE DATA SET NAME AND ROWS CONSTRAINTS CARDS

```
WRITE(6,700)
700 FORMAT('NAME',T15,'TEST')
WRITE(6,701)
701 FORMAT('ROWS')
WRITE(6,702)
702 FORMAT(T2,'N OBJF')
DO 200 J=1,N
IF(J.GT.9) GO TO 350
WRITE(6,703) J
703 FORMAT(T2,'G A0','I1)
GO TO 200
350 WRITE(6,753) J
753 FORMAT(T2,'G A0','I2)
200 CONTINUE
DO 201 I = 1, M
DO 201 J = 1, N
IF(I.LE.9.AND.J.LE.9) GO TO 351
IF(I.LE.9.AND.J.GT.9) GO TO 352
IF(I.GT.9.AND.J.LE.9) GO TO 353
IF(I.GT.9.AND.J.GT.9) GO TO 354
351 WRITE(6,741) I,J
741 FORMAT(T2,'L A',I1,',',I1)
GO TO 201
352 WRITE(6,742) I,J
742 FORMAT(T2,'L A',I1,',',I2)
GO TO 201
353 WRITE(6,743) I,J
743 FORMAT(T2,'L A',I2,',',I1)
GO TO 201
354 WRITE(6,744) I,J
744 FORMAT(T2,'L A',I2,',',I2)
201 CONTINUE
DO 202 I = 1, M
IF(I.GT.9) GO TO 320
WRITE(6,705) I
705 FORMAT(T2,'E AR',I1)
GO TO 202
320 WRITE(6,720) I
720 FORMAT(T2,'E AR',I2)
202 CONTINUE
```

PUNCH OUT THE COLUMNS DATA CARDS

```
WRITE(6,706)
```



```

706  FORMAT('COLUMNS')
      JX = 0
      JEND = M*N
      DO 300 J = 1,JEND
      JX = JX + 1
      DO 302 K = 1,M
      IF(J.LE.K*N,AND,J.GT.(K-1)*N) IX = K-1
      IF(J.EQ.IX*N+1) JX = 1
302  CONTINUE
      DO 301 I = 1,ILIM
      IF(KEY(I,J).NE.1) GO TO 301
      M2 = M + 2
      DO 303 L = 1,M2
      IF(I.LE.L*N,AND,I.GT.(L-1)*N) IA = L-1
303  CONTINUE
      JA = I - IA*N
      IF(I.GT.(M+1)*N) JA = I - (M+1)*N
      IF(IX.LE.9,AND,JX.LE.9,AND,IA.LE.9,AND,JA.LE.9) GO TO
1360 IF(IX.LE.9,AND,JX.GT.9,AND,IA.LE.9,AND,JA.LE.9) GO TO
1361 IF(IX.LE.9,AND,JX.GT.9,AND,IA.LE.9,AND,JA.GT.9) GO TO
1362 IF(IX.LE.9,AND,JX.GT.9,AND,IA.GT.9,AND,JA.LE.9) GO TO
1363 IF(IX.LE.9,AND,JX.GT.9,AND,IA.GT.9,AND,JA.GT.9) GO TO
1364 IF(IX.GT.9,AND,JX.GT.9,AND,IA.LE.9,AND,JA.GT.9) GO TO
1365 IF(IX.GT.9,AND,JX.GT.9,AND,IA.GT.9,AND,JA.LE.9) GO TO
1366 IF(IX.GT.9,AND,JX.GT.9,AND,IA.GT.9,AND,JA.GT.9) GO TO
1367 IF(IX.LE.9,AND,JX.LE.9,AND,IA.LE.9,AND,JA.GT.9) GO TO
1368 IF(IX.GT.9,AND,JX.LE.9,AND,IA.GT.9,AND,JA.LE.9) GO TO
1369 IF(IX.LE.9,AND,JX.LE.9,AND,IA.GT.9,AND,JA.GT.9) GO TO
1370 IF(IX.GT.9,AND,JX.LE.9,AND,IA.GT.9,AND,JA.GT.9) GO TO
1371 IF(IX.LE.9,AND,JX.LE.9,AND,IA.GT.9,AND,JA.LE.9) GO TO
1372 IF(IX.GT.9,AND,JX.LE.9,AND,IA.LE.9,AND,JA.GT.9) GO TO
1373 IF(IX.GT.9,AND,JX.GT.9,AND,IA.LE.9,AND,JA.LE.9) GO TO
1374 IF(IX.GT.9,AND,JX.LE.9,AND,IA.LE.9,AND,JA.LE.9) GO TO
1375 IF(I.GT.(M+1)*N) GO TO 380
      WRITE(6,760) IX,JX,IA,JA,AX(I,J)
760  FORMAT(T5,'X',I1,',',',',I1,T15,'A',I1,',',',',I1,T25,F10.7)
      GO TO 301
380  WRITE(6,780) IX,JX,JA,AX(I,J)
780  FORMAT(T5,'X',I1,',',',',I1,T15,'AR',I1,T25,F10.7)
      GO TO 301
361  IF(I.GT.(M+1)*N) GO TO 381
      WRITE(6,761) IX,JX,IA,JA,AX(I,J)
761  FORMAT(T5,'X',I1,',',',',I2,T15,'A',I1,',',',',I1,T25,F10.7)
      GO TO 301
381  WRITE(6,781) IX,JX,JA,AX(I,J)
781  FORMAT(T5,'X',I1,',',',',I2,T15,'AR',I1,T25,F10.7)
      GO TO 301
362  IF(I.GT.(M+1)*N) GO TO 382
      WRITE(6,762) IX,JX,IA,JA,AX(I,J)
762  FORMAT(T5,'X',I1,',',',',I2,T15,'A',I1,',',',',I2,T25,F10.7)
      GO TO 301
382  WRITE(6,782) IX,JX,JA,AX(I,J)
782  FORMAT(T5,'X',I1,',',',',I2,T15,'AR',I2,T25,F10.7)
      GO TO 301
363  IF(I.GT.(M+1)*N) GO TO 383
      WRITE(6,763) IX,JX,IA,JA,AX(I,J)

```



```

763 FORMAT(T5,'X',I1,',',',I2,T15,'A',I2,',',',I1,T25,F10.7)
GO TO 301
383 WRITE(6,783) IX,JX,JA,AX(I,J)
783 FORMAT(T5,'X',I1,',',',I2,T15,'AR',I1,T25,F10.7)
GO TO 301
364 IF(I. GT. (M+1)*N) GO TO 384
WRITE(6,764) IX,JX,IA,JA,AX(I,J)
764 FORMAT(T5,'X',I1,',',',I2,T15,'A',I2,',',',I2,T25,F10.7)
GO TO 301
384 WRITE(6,784) IX,JX,JA,AX(I,J)
784 FORMAT(T5,'X',I1,',',',I2,T15,'AR',I2,T25,F10.7)
GO TO 301
365 IF(I. GT. (M+1)*N) GO TO 385
WRITE(6,765) IX,JX,IA,JA,AX(I,J)
765 FORMAT(T5,'X',I2,',',',I2,T15,'A',I1,',',',I2,T25,F10.7)
GO TO 301
385 WRITE(6,785) IX,JX,JA,AX(I,J)
785 FORMAT(T5,'X',I2,',',',I2,T15,'AR',I2,T25,F10.7)
GO TO 301
366 IF(I. GT. (M+1)*N) GO TO 386
WRITE(6,766) IX,JX,IA,JA,AX(I,J)
766 FORMAT(T5,'X',I2,',',',I2,T15,'A',I2,',',',I1,T25,F10.7)
GO TO 301
386 WRITE(6,786) IX,JX,JA,AX(I,J)
786 FORMAT(T5,'X',I2,',',',I2,T15,'AR',I1,T25,F10.7)
GO TO 301
367 IF(I. GT. (M+1)*N) GO TO 387
WRITE(6,767) IX,JX,IA,JA,AX(I,J)
767 FORMAT(T5,'X',I2,',',',I2,T15,'A',I2,',',',I2,T25,F10.7)
GO TO 301
387 WRITE(6,787) IX,JX,JA,AX(I,J)
787 FORMAT(T5,'X',I2,',',',I2,T15,'AR',I2,T25,F10.7)
GO TO 301
368 IF(I. GT. (M+1)*N) GO TO 388
WRITE(6,768) IX,JX,IA,JA,AX(I,J)
768 FORMAT(T5,'X',I1,',',',I1,T15,'A',I1,',',',I2,T25,F10.7)
GO TO 301
388 WRITE(6,788) IX,JX,JA,AX(I,J)
788 FORMAT(T5,'X',I1,',',',I1,T15,'AR',I2,T25,F10.7)
GO TO 301
369 IF(I. GT. (M+1)*N) GO TO 389
WRITE(6,769) IX,JX,IA,JA,AX(I,J)
769 FORMAT(T5,'X',I2,',',',I1,T15,'A',I2,',',',I1,T25,F10.7)
GO TO 301
389 WRITE(6,789) IX,JX,JA,AX(I,J)
789 FORMAT(T5,'X',I2,',',',I1,T15,'AR',I1,T25,F10.7)
GO TO 301
370 IF(I. GT. (M+1)*N) GO TO 390
WRITE(6,770) IX,JX,IA,JA,AX(I,J)
770 FORMAT(T5,'X',I1,',',',I1,T15,'A',I2,',',',I2,T25,F10.7)
GO TO 301
390 WRITE(6,790) IX,JX,JA,AX(I,J)
790 FORMAT(T5,'X',I1,',',',I1,T15,'AR',I2,T25,F10.7)
GO TO 301
371 IF(I. GT. (M+1)*N) GO TO 391
WRITE(6,771) IX,JX,IA,JA,AX(I,J)
771 FORMAT(T5,'X',I2,',',',I1,T15,'A',I2,',',',I2,T25,F10.7)
GO TO 301
391 WRITE(6,791) IX,JX,JA,AX(I,J)
791 FORMAT(T5,'X',I2,',',',I1,T15,'AR',I2,T25,F10.7)
GO TO 301
372 IF(I. GT. (M+1)*N) GO TO 392
WRITE(6,772) IX,JX,IA,JA,AX(I,J)
772 FORMAT(T5,'X',I1,',',',I1,T15,'A',I2,',',',I1,T25,F10.7)
GO TO 301
392 WRITE(6,792) IX,JX,JA,AX(I,J)
792 FORMAT(T5,'X',I1,',',',I1,T15,'AR',I1,T25,F10.7)
GO TO 301
373 IF(I. GT. (M+1)*N) GO TO 393
WRITE(6,773) IX,JX,IA,JA,AX(I,J)
773 FORMAT(T5,'X',I2,',',',I1,T15,'A',I1,',',',I2,T25,F10.7)
GO TO 301

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```

393 WRITE(6,793) IX,JX,JA,AX(I,J)
793 FORMAT(T5,'X',I2,',',',',I1,T15,'AR',I2,T25,F10.7)
GO TO 301
374 IF(I. GT. (M+1)*N) GO TO 394
WRITE(6,774) IX,JX,IA,JA,AX(I,J)
774 FORMAT(T5,'X',I2,',',',',I2,T15,'A',I1,',',',',I1,T25,F10.7)
GO TO 301
394 WRITE(6,794) IX,JX,JA,AX(I,J)
794 FORMAT(T5,'X',I2,',',',',I2,T15,'AR',I1,T25,F10.7)
GO TO 301
375 IF(I. GT. (M+1)*N) GO TO 395
WRITE(6,775) IX,JX,IA,JA,AX(I,J)
775 FORMAT(T5,'X',I2,',',',',I1,T15,'A',I1,',',',',I1,T25,F10.7)
GO TO 301
395 WRITE(6,795) IX,JX,JA,AX(I,J)
795 FORMAT(T5,'X',I2,',',',',I1,T15,'AR',I1,T25,F10.7)
301 CONTINUE
IF(J. LT. JEND) GO TO 300
DO 305 IZ = 1,M
JZ = IZ
IF(IZ. GT. 9) GO TO 304
WRITE(6,709) IZ, JZ
709 FORMAT(T5,'XA',I1,T15,'OBJF',T25,'1.',T40,'AR',I1,T50,
1'1.')
GO TO 305
304 WRITE(6,708) IZ, JZ
708 FORMAT(T5,'XA',I2,T15,'OBJF',T25,'1.',T40,'AR',I2,T50,
1'1.')
305 CONTINUE
300 CONTINUE

```

PUNCH OUT THE RIGHT HAND SIDE DATA CARDS

```

710 WRITE(6,710)
FORMAT('RHS')
R DO 325 I = 1,N
R IF(I. GT. 9) GO TO 333
R WRITE(6,717) I, PX(I,1)
R 717 FORMAT(T5,'B1',T15,'A0',',',I1,T25,F10.7)
R GO TO 325
R 333 WRITE(6,718) I, PX(I,1)
R 718 FORMAT(T5,'B1',T15,'A0',',',I2,T25,F10.7)
R 325 CONTINUE
DO 306 I = 1,N
IF(M. LE. 9. AND. I. LE. 9) GO TO 311
IF(M. LE. 9. AND. I. GT. 9) GO TO 312
IF(M. GT. 9. AND. I. LE. 9) GO TO 313
IF(M. GT. 9. AND. I. GT. 9) GO TO 314
311 WRITE(6,711) M, I, X(I,1)
711 FORMAT(T5,'B1',T15,'A',I1,',',',',I1,T25,F10.7)
GO TO 306
312 WRITE(6,722) M, I, X(I,1)
722 FORMAT(T5,'B1',T15,'A',I1,',',',',I2,T25,F10.7)
GO TO 306
313 WRITE(6,723) M, I, X(I,1)
723 FORMAT(T5,'B1',T15,'A',I2,',',',',I1,T25,F10.7)
GO TO 306
314 WRITE(6,724) M, I, X(I,1)
724 FORMAT(T5,'B1',T15,'A',I2,',',',',I2,T25,F10.7)
306 CONTINUE
DO 307 I = 1,M
IF(I. GT. 9) GO TO 315
WRITE(6,712) I
712 FORMAT(T5,'B1',T15,'AR',I1,T25,'1.')
GO TO 307
315 WRITE(6,725) I
725 FORMAT(T5,'B1',T15,'AR',I2,T25,'1.')
307 CONTINUE
WRITE(6,713)
713 FORMAT('ENDATA')
801 IF(ITEST. EQ. IR1) GO TO 999

```



PUNCH OUT THE APPROPRIATE RHS REVISION DATA CARDS

```

      IF( ITEST.GT.9) GO TO 308
      WRITE(6,714) ITEST
714   FORMAT('NAME',T15,'REV',I1)
      GO TO 309
308   WRITE(6,715) ITEST
715   FORMAT('NAME',T15,'REV',I2)
309   WRITE(6,716)
      WRITE(6,716)
716   FORMAT(T3,'MODIFY')
      IT1 = ITEST + 1
R      DO 326 I = 1,N
R      IF(I.GT.9) GO TO 334
R      WRITE(6,717) I,PX(I,IT1)
R      GO TO 326
R 334  WRITE(6,718) I, PX(I,IT1)
R 326  CONTINUE
      DO 310 I = 1,N
      IF(M.LE.9.AND.I.LE.9) GO TO 321
      IF(M.LE.9.AND.I.GT.9) GO TO 322
      IF(M.GT.9.AND.I.LE.9) GO TO 323
      IF(M.GT.9.AND.I.GT.9) GO TO 324
321   WRITE(6,711) M, I, X(I,IT1)
      GO TO 310
322   WRITE(6,722) M, I, X(I,IT1)
      GO TO 310
323   WRITE(6,723) M, I, X(I,IT1)
      GO TO 310
324   WRITE(6,724) M, I, X(I,IT1)
310   CONTINUE
      WRITE(6,713)
      ITEST = ITEST + 1
      GO TO 801
999   CONTINUE
      STOP
      END

```

\* \* \* \* \*



```

* * * * *
* * DATA GENERATOR - TEST FOR A REACHABLE SET IN M STEPS * *
* * * * * ROBERT L. ARMACOST - - - 1 JULY 1970 * * * * *
* * * * *
* * * * *

```

THIS PROGRAM WILL PUNCH OUT A SET OF DATA CARDS FOR A LINEAR PROGRAM USING THE MPS PACKAGE ON THE IBM 360/67. THE BASIC INPUT TO THE PROGRAM IS A SQUARE PROMOTION MATRIX OF SIZE N, AND K VECTORS WHICH ARE TO BE TESTED TO DETERMINE IF THEY ARE ELEMENTS OF THE SET  $B\langle M \rangle$ . THE PROGRAM WILL GENERATE DATA DECKS FOR A SERIES OF REACHABLE SETS. MBEG IS THE STARTING INDEX FOR THE SUBSCRIPT OF THE TEST. MEND IS THE ENDING INDEX FOR THE SUBSCRIPT OF THE TEST. THESE INDICES ARE READ IN AS DATA. THE FIRST VECTOR TO BE TESTED IS CALLED "TEST". EACH SUCCEEDING VECTOR IS CALLED "REVI" WHERE I IS THE NUMBER OF THE REVISION OF THE RIGHT HAND SIDE. THE VARIABLE IR IS THE NUMBER OF REVISIONS IN THE CONTROL PROGRAM. THE ARRAYS SHOULD BE DIMENSIONED IN THE FOLLOWING WAY -

```

      AX((MEND)*(N+1) + N, MEND*(N+1))
      KEY - SAME AS AX
      X(N,IR+1)
      P(N,N)
      PT(N,N)
      PTI(N,N).

```

THE PROGRAM WILL ACCCOMODATE A PROMOTION MATRIX WITH N = 99 AND WITH MEND < 100.

```

* * * * *
* * * * *

```

THIS PROGRAM IS CONSTRUCTED BY REMOVING THOSE CARDS MARKED WITH AN R IN PROGRAM TO TEST FOR RETURN IN M STEPS, AND TO REPLACE THOSE MODULES MARKED WITH NUMBERS BY THE MODULES LISTED BELOW WITH THE CORRESPONDING NUMBERS.

```

* * * * *

```

```

1      READ(5,600) MBEG, MEND, N, IR
2      CALCULATE THE TRANSPOSE OF THE PROMOTION MATRIX, PT,
2      AND CALCULATE I - PT WHERE I IS THE IDENTITY MATRIX
2
2      DO 20 I=1,N
2      DO 20 J=1,N
2      PT(I,J) = P(J,I)
2      PTI(I,J) = -PT(I,J)
2      IF(I.EQ.J) PTI(I,J) = 1. + PTI(I,J)
2 20 CONTINUE
3
3      INSERT PTI IN THE FIRST N ROWS AND COLUMNS OF THE
3      CONSTRAINT MATRIX, AX

```



```

3      DO 30 I=1,N
3      DO 30 J=1,N
3      AX(I,J) = PTI(I,J)
3  30  IF(AX(I,J).NE.0) KEY(I,J) = 1
4      (M-1)*N ROWS OF AX

```

```

* * * * *

```



\* \* \* \* \*

\* STEADY STATE SET - EXTREME POINTS AND TEST FOR INCLUSION \*

\* \* \* \* \* ROBERT L. ARMACOST - - - 1 JULY 1970 \* \* \* \* \*

\* \* \* \* \*

THE MAIN PROGRAM IS DIMENSIONED TO HANDLE A PROMOTION MATRIX WITH 15 STATES. THE PROMOTION MATRIX IS P AND ITS INVERSE WHICH HAS BEEN NORMALIZED IS PIN. THE ROWS OF PIN ARE THE EXTREME POINTS OF THE STEADY STATE SET.

THE EXTREME POINTS OF THE STEADY STATE SET ARE USED TO DETERMINE IF A TEST POINT IS IN THE STEADY STATE SET BY CALCULATING THE NECESSARY COEFFICIENTS TO MAKE THE TEST POINT A LINEAR COMBINATION OF THE EXTREME POINTS. IF THE SUM OF THE COEFFICIENTS IS GREATER THAN ONE, OR IF ANY COEFFICIENT IS NEGATIVE, WE CONCLUDED THAT THE TEST POINT IS NOT IN THE STEADY STATE SET. THE COEFFICIENTS ARE DENOTED ALF, AND THE TEST POINTS ARE X. THE PROGRAM IS SET UP TO ACCEPT FIVE SUCH TEST VECTORS WITH FIFTEEN RANKS.

THE SIZE OF THE PROMOTION MATRIX AND THE LENGTH OF THE TEST VECTOR ARE INPUT TO THE PROGRAM IN THE VARIABLE NN. THE NUMBER OF TEST VECTORS IS INPUT IN THE VARIABLE KK.

\* \* \* \* \*

```

1  DIMENSION UNI(15,15),P(15,15), PIN(15,15), L(15),
599  1M(15), SUM(15), ALF(15), X(15,5)
    READ(5,599) NN, KK
    FORMAT(2I5)
    CALL WORK(UNI,P,PIN,L,M,ALF,X,NN,KK,SUM)
    CALL EXIT
    END

```

\* \* \* \* \*

\* \* \* SUBROUTINE WORK - THIS SUBROUTINE CALCULATES THE \* \* \*  
 \* \* \* EXTREME POINTS OF THE STEADY STATE SET, M, AND \* \* \*  
 \* \* \* TESTS EACH TEST VECTOR FOR INCLUSION IN THAT SET \* \* \*

```

    SUBROUTINE WORK(UNI,P,PIN,L,M,ALF,X,NN,KK,SUM)
    DIMENSION UNI(NN,NN),P(NN,NN),PIN(NN,NN),L(NN),
600  1M(NN), SUM(NN), ALF(NN),X(NN,KK)
    READ(5,600) ((P(I,J),J=1,NN),I=1,NN)
    FORMAT(8F10.0)
    READ(5,601) ((X(J,K),J=1,NN),K=1,KK)
601  FORMAT(8F10.0)

```

INITIALIZE

```

DO 10 I = 1,NN
  L(I) = 0
  M(I) = 0

```



```

SUM(I) = 0.
DO 10 J = 1, NN
  UNI(I, J) = - P(I, J)
  IF(I.EQ.J) UNI(I, J) = 1. + UNI(I, J)
10 CONTINUE
  WRITE(6, 699)

WRITE OUT PROMOTION MATRIX

DO 2 I=1, NN
  WRITE(6, 700) (P(I, J), J=1, NN)
  WRITE(6, 710)
2 CONTINUE

CALCULATE THE INVERSE OF P
CALL MINV(UNI, NN, D, L, M)

CALCULATE THE NORMALIZED INVERSE OF P, PIN

DO 20 I=1, NN
DO 15 J=1, NN
  SUM(I) = SUM(I) + UNI(I, J)
15 CONTINUE
DO 25 J=1, NN
  PIN(I, J) = UNI(I, J) / SUM(I)
25 CONTINUE
20 CONTINUE
  WRITE(6, 699)
699 FORMAT(1H1)

WRITE OUT NORMALIZED INVERSE OF P

DO 5 I=1, NN
  WRITE(6, 700) (PIN(I, J), J=1, NN)
  WRITE(6, 710)
5 CONTINUE
700 FORMAT(15F8.6)
710 FORMAT(1H )

TEST TO SEE IF TEST VECTOR IS IN THE STEADY STATE SET

DO 130 K=1, KK
  MM=K
  DO 1 I=1, NN
    ALF(I) = 0.
    ALF(I) = X(1, K) / PIN(1, 1)
    TEST = ALF(I)
    JEND = NN-1
    DO 120 J=2, JEND
      IEND = J - 1
      SUMD = 0.
      DO 110 I=1, IEND
        SUMD = SUMD + ALF(I) * PIN(I, J)
        ALF(J) = (X(J, K) - SUMD) / PIN(J, J)
        TEST = TEST + ALF(J)
        IF(ALF(J).LT.0) GO TO 100
        IF(TEST.GT.1) GO TO 200
      120 CONTINUE
      ALF(NN) = 1. - TEST
      IF(ALF(NN).LT.0) GO TO 100
      IF(ALF(NN).LT.X(NN, K)) GO TO 200
    130 CONTINUE
    WRITE(6, 702)
    702 FORMAT(1H1, T20, 'THE VECTOR IS IN THE STEADY STATE SET'
    1 // T20, 'ALF COEFFICIENTS', T45, 'TEST VECTOR' //)
    DO 201 LL=1, NN
      WRITE(6, 703) ALF(LL), X(LL, MM)
    703 FORMAT(T20, F10.8, T45, F10.8)
    201 CONTINUE
    IF(K.LT.KK) GO TO 130
    GO TO 999

```



```

100 WRITE(6,704)
704 FORMAT(1H1,T20,'THE VECTOR IS NOT IN THE STEADY STATE'
1,' SET, ALF IS NEGATIVE'//T20,'ALF COEFFICIENTS',T45,
2'TEST VECTOR'//)
DO 202 LL=1,NN
WRITE(6,703) ALF(LL), X(LL,MM)
202 CONTINUE
IF(K.LT.KK) GO TO 130
GO TO 999
200 WRITE(6,705)
705 FORMAT(1H1,T20,'THE VECTOR IS NOT IN THE STEADY STATE'
1,' SET, TEST IS GREATER THAN ONE'//T20,
2'ALF COEFFICIENTS',T45,'TEST VECTOR'//)
DO 203 LL=1,NN
WRITE(6,703) ALF(LL), X(LL,MM)
203 CONTINUE
IF(K.LT.KK) GO TO 130
999 RETURN
END

```

\*\*\*\*\*



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